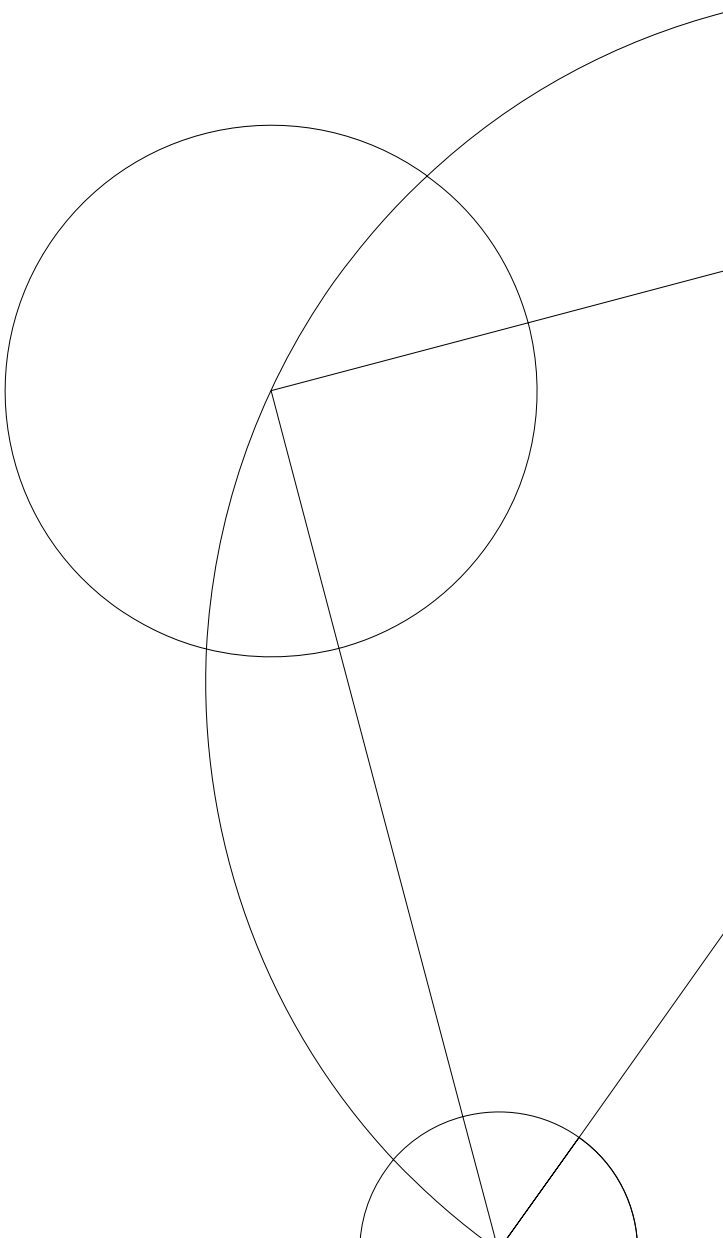
DEPARTMENT OF COMPUTER SCIENCE
UNIVERSITY OF COPENHAGEN

Functional Object Analysis

Toward Statistical Analysis of Functional Objects



PhD thesis

Lars Lau Raket

Thesis overview

This document constitutes my PhD thesis at Department of Computer Science, University of Copenhagen. The subject of the thesis is statistical analysis of functional data, and the main goal of the enclosed work is to go beyond the typical simple analyses of curve data, and open up the possibility of doing model-based statistical analysis of complex functional objects. The functional aspect of data naturally complicate statistical analysis; as opposed to conventional data analysis, geometric information has to be taken into account, and potentially enormous data sizes has to be handled. The contributions of this thesis are both theoretical, computational, and practical, and it offers solutions to some of the mentioned problems in various relevant cases.

During my PhD studies I have worked on a variety of topics. In particular, I have done much work on optical flow estimation and distributed video coding. Rather than writing an overview of my research output during my PhD studies, and the spurious relations between the different work, I have chosen to write a thesis on the area which I find most interesting, and on which I will continue my future research. Thus, optical flow estimation and video coding will not be considered in much detail in this thesis.

Lars Lau Raket
26 March 2014

Included papers

The main contribution of this thesis are the three papers

L.L. Rakêt and B. Markussen, "Approximate inference for spatial functional data on massively parallel processors," *Computational Statistics & Data Analysis*, vol. 72, pp. 227-240, 2014.

L.L. Rakêt, S. Sommer, and B. Markussen, "A nonlinear mixed-effects model for simultaneous smoothing and registration of functional data" *Pattern Recognition Letters*, vol. 38, pp. 1-7, 2014.

L.L. Rakêt, B. Grimme, B. Markussen, G. Schöner, and C. Igel, "Statistical analysis of

human arm movements using timing and motion separation," working paper, 2014.

Finally, Appendix ?? includes a description of an image registration algorithm, which is based on results from the works

L.L. Rakêt, *Duality based optical flow algorithms with applications,* University of Copenhagen prize thesis in Computer Science, Copenhagen University Library, 2013.

L.L. Rakêt, L. Roholm, M. Nielsen, and F. Lauze, "TV-L^1 optical flow for vector valued images," in *Energy Minimazation Methods in Computer Vision and Pattern Recognition* (Y. Boykov, F. Kahl, V. Lempitsky, and F. Schmidt, eds.), vol. 6819 of *Lecture Notes in Computer Science*, pp. 329-341, Springer, 2011.

Other scientific publications during the PhD study

While I have found that statistical analysis of functional data has been my most exciting contribution, I have spent the majority of my PhD working in other areas. I have made a number contributions to the fields of image and video processing, where I have mainly focused on variational formulations. Furthermore, I have done a considerable amount of work on integrating such variational methods in video codecs in the field of distributed video coding. These endeavors have been quite successful—for example the MORE codec (Luong et al., 2014), that I have recently co-developed, is the best performing single-view distributed video codec to date. The papers describing this work do unfortunately not fit into the topic of the present thesis, and have therefore been excluded.
The list of excluded publications are as follows:

M. Salmistraro, **L.L. Rakêt**, C. Brites, J. Ascenso, and S. Forchhammer, "Joint disparity and motion estimation using optical flow for multiview distributed video coding", *European Signal Processing Conference (EUSIPCO)* (**accepted**), 2014.

J. Petersen, M.M.W. Wille, **L.L. Rakêt**, A. Feragen, J.H. Pedersen, M. Nielsen, A. Dirksen, M. de Bruijne, "Effect of inspiration on airway dimensions measured in maximal inspiration CT images", *European Radiology* (**in press**), 2014.

H.V. Luong, **L.L. Rakêt**, and S. Forchhammer, "Re-estimation of motion and reconstruction for distributed video coding," *Image Processing, IEEE Transaction on*, vol. 23, pp. 2804-2819 2014.

M. Salmistraro, **L.L. Rakêt**, M. Zamarin, A. Ukhanova, and S. Forchhammer, "Texture side information generation for distributed coding of video-plus-depth," in *Image Processing (ICIP), 2013 20th IEEE International Conference on,* pp. 1699-1703, 2013.

M. Salmistraro, M. Zamarin, **L.L. Rakêt**, and S. Forchhammer, "Distributed multi-hypothesis coding of depth maps using texture motion information and optical flow," in *Acoustics, Speech and Signal Processing (ICASSP), 2013 IEEE International Conference on,* pp. 1685-1689, 2013.

H.V. Luong, **L.L. Rakêt**, X. Huang, and S. Forchhammer, "Side information and noise learning for distributed video coding using optical flow and clustering," *Image Processing, IEEE Transaction on*, vol. 21, pp. 4782-4796, 2012.

L.L. Rakêt and M. Nielsen, "A splitting algorithm for directional regularization and sparsification," in *Pattern Recognition (ICPR), 2012 21st International Conference on,* pp. 3094-3098, 2012.

L.L. Rakêt, "Local smoothness for global optical flow," in *Image Processing (ICIP), 2012 19th IEEE International Conference on,* pp. 1-4, 2012.

L.L. Rakêt, J. Søgaard, M. Salmistraro, H.V. Luong, and S. Forchhammer, "Exploiting the error-correcting capabilities of low density parity check codes in distributed video coding using optical flow," in *Proceedings of SPIE, the International Society for Optical Engineering*, vol. 8499 SPIE – International Society for Optical Engineering, 2012.

L.L. Rakêt, L. Roholm, A. Bruhn, and J. Weickert, "Motion compensated frame interpolation with a symmetric optical flow constraint," in *Advances in Visual Computing* (G. Bebis et al., eds.), vol. 7431 of *Lecture Notes in Computer Science*, pp. 329-341, Springer, 2012.

X. Huang, **L.L. Rakêt**, H.V. Luong, M. Nielsen, F. Lauze, and S. Forchhammer, "Multi-hypothesis transform domain Wyner-Ziv video coding including optical flow," in *Multimedia Signal Processing (MMSP), 2011 IEEE 13th International Workshop on*, pp. 1-6, 2011.

Summary

I propose a direction it the field of statistics which I denote *functional object analysis*. This subfields considers the analysis of functional objects defined on continuous domains. In this setting I will focus on model-based statistics, with a particularly emphasis on mixed-effect formulations, where the observed functional signal is assumed to consist of both fixed and random functional effects. This thesis takes the initial steps toward the development of likelihood-based methodology for functional objects. I first consider analysis of functional data defined on high-dimensional Euclidean spaces under the effect of additive spatially correlated effects, and then move on to consider how to include data alignment in the statistical model as a nonlinear effect under additive correlated noise. In both cases, I will give directions on how to generalize the methodology to more complex data setups. Finally, I consider extensions and future directions.

Contributions

The main methodological contributions of this thesis are as follows:

- An operator approximation framework for doing inference in linear functional mixed-effects models (Raket and Markussen, 2014).
- A description, and source code for doing efficient massively parallel inference in functional mixed-effects models (Raket and Markussen, 2014).
- A model for doing likelihood inference for functional data under the effect of, possibly random, alignment variation (Raket et al., 2014).

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